Inexpressible Longing For The Intended Model

Summary

The *intended model* of a given theory is a model distinguished among all models of this theory by the fact that the theory was built in order to describe exactly this model. It is thus a *pragmatic* concept. Models may be indistinguishable with respect to *structure* (i.e. *isomorphic*) or with respect to *semantic* properties (e.g. *elementarily equivalent*). Some metalogical facts (*Compactness Theorem*, *Löwenheim-Skolem Theorem*) imply negative results concerning *categoricity* of consistent theories (in a first order language). It also follows from the well known *Incompleteness Theorems* that many important theories are not *complete*, i.e. they do have models which are not elementarily equivalent. Classical and modern model theory provides numerous theorems concerning *categoricity in power* and its connections with semantic properties of theories (e.g. *Ryll-Nardzewski Theorem*, *Morley Theorem*). It is also obvious that properties related to categoricity and completeness are dependent upon the logic chosen, i.e. upon *expressive power* of a logical system.

Looking for a precise characterization of intended models is connected also with e.g.: *representation theorems*, some theorems concerning non standard models (as for example *Tennenbaum's Theorem*), and *extremal axioms*. Well known examples of extremal axioms are, among others: Hilbert's *Completeness Axiom* in geometry, Fraenkel's *Axiom of Restriction* in set theory, Gödel's *Axiom of Constructibility*, Suszko's *Axiom of Canonicity*, numerous *axioms of existence of very large cardinal numbers*, *Axiom of Induction* in arithmetic. They were proposed with the hope that they could catch the meaning of the concept of an intended model.

In the lecture we will try to give a synopsis of the above results. We are interested first of all in mathematical theories, but some examples from empirical theories will be given, too. We promise to avoid a complicated mathematical formalism. Finally, we will mention a few philosophical problems concerning intended models.

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