Role of infinity in computer science

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In classical computability theory, computation is viewed as a process to get results. The quality of algorithmic solutions is measured by reliability and speed. The situation is different for systems designed to act continuously like, e.g., operational systems. Here the entire behaviour of a system – ideally infinite – is subject to evaluation. Automata on infinite objects – words or trees, originally introduced by Buchi and Rabin to prove decidability of some fragments of Peano arithmetic, have subsequently proved to be a useful mathematical tool to analyse such systems. However, infinite words or trees, sometimes in relation with real numbers, have occurred to the mind of mathematicians much earlier, as early as the beginning of the XXth century. A discovery of Suslin and Lusin that the operation of projection may take us beyond the universum of Borel sets gave rise to a new branch of mathematics: descriptive set theory. It is intriguing that "difficult" sets discovered in that context have later turned out to be typical in the analysis of computer systems. The aim of the lecture is to present mutual inspiration between descriptive set theory and automata theory, in particular in relation to "difficulty" of sets. The measures of difficulty elaborated in these theories sometimes agree, but sometimes diverge, perhaps leading to a yet another, more general, theory.