# Three Lines of Investigation at the Metaphysics Research Lab

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# The Structure of the Talk

- Review the Axiomatic Theory of Abstract Objects
- The Philosophy of Mathematics, Neologicism, and Logicism
- An Epistemology for Abstract Objects
- Computational Metaphysics

# Review of the Axiomatic Theory of Abstract Objects

# The Theory of Abstract Objects I: Language

- Object variables and constants: $x, y, z, \ldots ; a, b, c, \ldots$
- Relation variables and constants: $F^n, G^n, H^n, \ldots ;$
  $P^n, Q^n, R^n, \ldots$ (when $n \geq 0$); $p, q, r, \ldots$ (when $n = 0$)
- Distinguished 1-place relation: $E!$ (read: *concrete*)
- Atomic formulas:
  $F^n x_1 \ldots x_n$ (‘$x_1, \ldots, x_n$ exemplify $F^n$’)
  $xF^1$ (‘$x$ encodes $F^1$’)
- Complex Formulas: $\neg\varphi$, $\varphi \rightarrow \psi$, $\forall\alpha\varphi$ ($\alpha$ any variable), $\Box\varphi$
- Complex Terms:
  Descriptions: $\iota x\varphi$
  $\lambda$-predicates: $[\lambda x_1 \ldots x_n\ \varphi]$ ($\varphi$ no encoding subformulas)

# The Theory of Abstract Objects: Definitions I

- &, $\vee$, $\equiv$, $\exists$, and $\Diamond$ are all defined in the usual way
- *Ordinary* objects are possibly concrete
  $O! =_{df} [\lambda x \Diamond E!x]$
- *Abstract* objects couldn't be concrete
  $A! =_{df} [\lambda x \neg\Diamond E!x]$
- $x$ and $y$ are E-identical iff $x$ and $y$ are both ordinary and necessarily exemplify the same properties
  $x =_E y =_{df} O!x \,\&\, O!y \,\&\, \Box\forall F(Fx \equiv Fy)$
- $x$ and $y$ are identical iff either $x$ and $y$ are E-identical or $x$ and $y$ are both abstract and necessarily encode the same properties
  $x = y =_{df} x =_E y \vee (A!x \,\&\, A!y \,\&\, \Box\forall F(xF \equiv yF))$

## The Theory of Abstract Objects: Definitions II

- $F$ and $G$ are identical iff $F$ and $G$ are necessarily encoded by the same objects
  $F^1 = G^1 \;=_{df}\; \Box\forall x(xF^1 \equiv xG^1)$

- $p$ and $q$ are identical iff the property *being such that p* is identical to property *being such that q*
  $p = q \;=_{df}\; [\lambda y\ p] = [\lambda y\ q]$

# The Theory of Abstract Objects: Logic

- Simplest second-order quantified S5 modal logic:
  1st and 2nd order Barcan formulas (i.e., fixed domains)
  (as in Linsky & Zalta 1994, Williamson 1998)

- Logic of Encoding: $\Diamond xF \rightarrow \Box xF$

- Logic of Identity: $\alpha = \beta \rightarrow [\varphi(\alpha, \alpha) \equiv \varphi(\alpha, \beta)]$
  ($\beta$ substitutable for $\alpha$)

- Classical Logic of $\lambda$-Predicates:
  $[\lambda x_1 \ldots x_n\ \varphi]y_1 \ldots y_n \equiv \varphi^{y_1,\ldots,y_n}_{x_1,\ldots,x_n}$ ($\varphi$ free of descriptions)
  e.g., $[\lambda x\ \neg Rx]y \equiv \neg Ry$

- Classical Logic of (Rigid) Descriptions

## The Theory Proper

A. Proper Axioms

- $O!x \rightarrow \Box\neg\exists F\, xF$
- $\exists x(A!x \,\&\, \forall F(xF \equiv \varphi))$, where $\varphi$ has no free $x$s

B. Well-Defined Descriptions

- $\imath x(A!x \,\&\, \forall F(xF \equiv \varphi))$

C. Proper Theorem Schema

- $\imath x(A!x \,\&\, \forall F(xF \equiv \varphi))G \;\equiv\; \varphi^G_F$

## Some Examples of Abstract Objects: I

- *The Complete Concept of y* = the abstract object that encodes exactly the properties $y$ exemplifies

  $$\imath x(A!x \,\&\, \forall F(xF \equiv Fy))$$

- *PossibleWorld*$(x)$ $=_{df}$ $x$ might be such that it encodes all and only true propositions

  $$\Diamond\forall p(x[\lambda y\; p] \equiv p)$$

  *p is true at w* ('$w \models p$') $=_{df}$ $w[\lambda y\; p]$

- *The Actual World* = the abstract object that encodes all and only true propositions

  $$\imath x(A!x \,\&\, \forall F(xF \equiv \exists p(p \,\&\, F = [\lambda y\; p])))$$

## Some Examples of Abstract Objects: II

- *The Truth Value of p* = the abstract object that encodes all and only the propositions $q$ materially equivalent to $p$

  $$\imath x(A!x \,\&\, \forall F(xF \equiv \exists q(q \equiv p \,\&\, F = [\lambda y\, q])))$$

- *The Extension of the Concept G* = the abstract object that encodes all and only the properties $F$ materially equivalent to $G$

  $$\imath x(A!x \,\&\, \forall F(xF \equiv \forall y(Fy \equiv Gy)))$$

- *The Form of G* = the abstract object that encodes all and only the properties $F$ necessarily implied by $G$

  $$\imath x(A!x \,\&\, \forall F(xF \equiv \Box\forall y(Gy \rightarrow Fy)))$$

# Philosophy of Mathematics, Neologicism and Logicism

# Neologicism: I

- Logicism: Mathematics is reducible to logic alone.
- Almost no defenders of logicism nowadays.
- There are 3 ways to weaken logicism, in the attempt to find a true thesis (holding mathematics fixed):
  1. expand the notion of logic in minimal ways
  2. allow limited non-logical resources, e.g., analytic truths
  3. revise the notion of reducibility
- The division of positions:
  1. Hodes (1984, 1991), Tenant (2004) follow (1)
  2. Wright (1983), Hale (1987, 2000), Hale & Wright (2001), Boolos (1986), Cook (2003), Fine (2002) follow (2)
  3. Zalta (2000), Linsky & Zalta (1995, 2006) follow (3).

## Neologicism: II

- Positions (1) and (2) face difficulties:
  - Some add axioms of infinity.
  - Some add logical principles in a piecemeal way.
  - Some face the Julius Caesar problem.
  - Some face bad-company/embarassment of riches problems.
  - Almost all add mathematical primitives.
- With one exception (Cook 2003), positions (1) and (2) run up against ‘the limits of abstraction’.
- Cook’s principles are so strong as to be mathematical rather than logical; he adds: axiom of infinity, new mathematical primitives (EXTs, ORDs), principles which aren’t even close to being analytic, a new kind of Julius-Caesar problem, etc.

## Neologicism: III

- We argue (Linsky & Zalta 2006): third-order object theory is a version of neologicism: it employs a new kind of reduction and has no limits of abstraction, since arbitrary mathematical theories can be reduced to third-order logic and analytic truths.

# Mathematical Objects

- *p is true in T* (‘$T \models p$’) $=_{df}$ $T[\lambda y\, p]$
  i.e., treat mathematical theories as objects that encode propositions
- For each formula $\varphi$ that is an axiom of $T$, add the analytic truth:
  $T \models \varphi^*$ (with primitive constants $\kappa$ in $\varphi$ replaced by $\kappa_T$ in $\varphi^*$)
- Closure Rule: If $p_1, \ldots, p_n \vdash q$ and $T \models p_1, \ldots, T \models p_n$,
  infer $T \models q$
- Reduction Axiom: Theoretically identify individual $\kappa_T$ as follows:

$$\kappa_T = \iota x(A!x \,\&\, \forall F(xF \equiv\ T \models F\kappa_T))$$

$$0_{\text{PNT}} = \iota x(A!x \,\&\, \forall F(xF \equiv\ \text{PNT} \models F0_{\text{PNT}}))$$

$$\emptyset_{\text{ZF}} = \iota x(A!x \,\&\, \forall F(xF \equiv\ \text{ZF} \models F\emptyset_{\text{ZF}}))$$

## Mathematical Relations

- Third-order language, comprehension over abstract properties/relations:

$$\Pi_{\mathrm{T}} \;=\; \iota R(\mathbf{A}!R \;\&\; \forall \mathbf{F}(R\mathbf{F} \equiv \textsc{t} \models \mathbf{F}\Pi_{\mathrm{T}}))$$

- In other words: the property $\Pi$ of theory $\textsc{t}$ is the abstract relation $R$ which encodes all and only those second-level properties $\mathbf{F}$ such that in theory $\textsc{t}$, $\Pi$ exemplifies $\mathbf{F}$.

- This does not *introduce* the relation $\Pi$ but rather is a principle that identifies $\Pi$ in terms of its role in $\textsc{t}$.

- Examples:

$$S_{\mathrm{PNT}} = \iota R(\mathbf{A}!R \;\&\; \forall \mathbf{F}(R\mathbf{F} \equiv \textsc{pnt} \models \mathbf{F}S_{\mathrm{PNT}}))$$

$$\in_{\mathrm{ZF}} = \iota R(\mathbf{A}!R \;\&\; \forall \mathbf{F}(R\mathbf{F} \equiv \textsc{zf} \models \mathbf{F}\in_{\mathrm{ZF}}))$$

## The Truth of Mathematical Sentences

- There are true (encoding) readings of ordinary mathematical statements (i.e., those with no 'theory-operator' prefixed).
- The sentence:

  In real number theory, $\sqrt{2}$ is algebraic
  RNT $\models A\sqrt{2}$ (dropping subscripts)

  is equivalent to '$\sqrt{2}A$'.
- So the ordinary mathematical sentence:

  $\sqrt{2}$ is algebraic

  is ambiguous between

  $\sqrt{2}A$ (true)
  $A\sqrt{2}$ (false)

# Logicism

- This becomes logicism once we replace comprehension:

  $$\exists x(A!x \,\&\, \forall F(xF \equiv \varphi))$$
  $$\exists R(\mathbf{A}!R \,\&\, \forall \mathbf{F}(R\mathbf{F} \equiv \varphi))$$

  by abstraction:

  $$\imath x(A!x \,\&\, \forall F(xF \equiv \varphi))G \equiv \varphi^{G}_{F}$$
  $$\imath R(\mathbf{A}!R \,\&\, \forall \mathbf{F}(R\mathbf{F} \equiv \varphi))\mathbf{G} \equiv \varphi^{\mathbf{G}}_{\mathbf{F}}$$

- Object theory uses: third-order logic (under general models!), the above analytic truths (which can be true in very small models), and analytic (prefaced by the 'in the theory' operator) truths of mathematics. This gives a new form of mathematical reduction:
  - A denotation for each well-defined term of an arbitrary theory T.
  - A true (encoding) reading for each theorem of T

# An Epistemology for Abstract Objects

# Epistemology: I

- A motivating force behind Frege's logicism: how do we grasp numbers?
- Linsky & Zalta 1995: to reconcile platonism and naturalism, form a proper conception of the mind-independence and objectivity of abstract objects.
- Don't use the epistemological model of physical objects:
  - Physical objects are sparsely spread out through their domain. One needs to do real work to discover them.
  - Physical objects are subject to an appearance/reality distinction. They have backsides!
  - They are complete and determinate, with a some exceptions.

# Epistemology: II

- Abstract objects are not like this:
  - They are governed by principle of comprehension, yielding a plenitude, not a sparse domain. There are as many as there could possibly be.
  - There is no appearance/reality distinction. They are just the way we take them to be in our descriptions of them.
  - There is a dimension in which they are incomplete.
- With this model, we argued that:
  - Knowledge of mathematical objects (and abstract objects in general) is by description: $\iota x(A!x \;\&\; \forall F(xF \equiv \varphi))$
  - All we have to do to have knowledge of an abstract object is to understand its description: knowledge by acquaintance and knowledge by description collapse.

# Epistemology: III

- This is consistent with the principles of naturalism:
  - Comprehension is unrestricted; abstract objects are postulated in non-piecemeal and non-arbitrary way.
  - It is parsimonius: we should accept as few abstract objects in a non-arbitrary way, but with abstract objects governed by a comprehension principle, the only way to accept as few as possible is to accept them all.
- Today: advance this epistemology in two new directions. (1) Appeal to the logicist epistemology, and (2) Reconceptualize the formalism, not as a kind of platonism, but as a principle that captures Wittgenstein's meaning as use doctrine.

## Logicist Epistemology

"But in reply to Kant, logicists claimed that these [mathematical] propositions are a priori because they are analytic—because they are true (false) merely 'in virtue of' the meanings of the terms in which they are cast. Thus to know their meanings is to know all that is required for a knowledge of their truth. No empirical investigation is needed. The philosophical point of establishing the view was nakedly epistemological: logicism, if it could be established, would show that our knowledge of mathematics could be accounted for by whatever would account for our knowledge of language. And, of course, it was assumed that knowledge of language could *itself* be accounted for in ways consistent with empiricist principles, that language was itself entirely learned. Thus, following Hume, all our knowledge could once more be seen as concerning either 'relations of ideas' (analytic and a priori) or 'matters of fact'." (Benacerraf 1981, 42–43)

## Reconceptualize Abstract Objects

- **Goal**: To find a conception of abstract objects as entities we all believe in as naturalists.
- **Method**: Use a 'bottom-up' interpretation of the comprehension principle.
- **Interpretation**: it systematizes mathematical practice, i.e., systematizes a variety of large-scale patterns of behavior (patterns of speech, language use, etc.) that mathematicians engage in when they do number theory, linear algebra, analysis, etc.

# The Reinterpretation: I

- In mathematical practice, mathematicians state principles, introducing new terms using the language of those principles, and proving new theorems from those principles. E.g., Zermelo laying down principles involving the terms '$\emptyset$', $\cup$, and the predicate '$\in$'
- Mathematicians use these terms to ground referential and anaphoric uses of the pronoun 'it', to formulate new claims, to prove such claims are consequences, etc., and in discussing the theory with one another.
- This is systematic, rule-governed behavior which has various uniformities.

# The Reinterpretation: II

- Comprehension and identification systematize this practice, by justifying the move from claims of the form 'In $\tau$, $\Pi\kappa$' to existential quantifications over the objects and properties of $\tau$.
- Comprehension is therefore a pattern-extractor, yielding a view of mathematical objects and relations on which they are not self-subsistent, but dependent on mathematical practice.
- Indeed, in the case of mathematics, you need the practice to instantiate the comprehension principle
- This makes a Wittgensteinian understanding of language more precise and helps to naturalize a piece of formal metaphysics.

# Computational Metaphysics

Car si nous l'avions telle que je la conçois, nous pourrions raisonner en metaphysique et en morale à pue pres comme en Geometrie et en Analyse Leibniz (Gerhardt 1890, vii, 21)

If we had it [a *characteristica universalis*], we should be able to reason in metaphysics and morals in much the same way as in geometry and analysis. Russell 1900, 169

Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo . . . dicere: calculemus.

Leibniz (Gerhardt 1890, vii, 200)

If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, to sit down to their slates, and to say to each other . . . : Let us calculate. Russell 1900, 170

# Implementation in PROVER9: I

- Basic Notation (PROVER9 syntax in parentheses):

| | | | |
|---|---|---|---|
| Predicates | $A, B, C$ (`A`, `B`, `C`) | Constants | $a, b, c$ (`a`, `b`, `c`) |
| Variables | $x, y, z$ (`x`, `y`, `z`) | Functions | $f, g, h$ (`f`, `g`, `h`) |
| Quantifiers | $\forall, \exists$ (NA) | Connectives | $\&, \rightarrow, \vee, \neg, =$ (NA, NA, `\|`, `-`, `=`) |

- Formulas *vs* Clauses (quantifier elimination and CNF)

| Formula | Clause (PROVER9 — $Q$-free, and CNF) |
|---|---|
| $(\forall x)(Px \rightarrow Qx)$ | `-P(x) \| Q(x).` |
| $(\exists x)(Px \,\&\, Qx)$ | `P(a). Q(a).` (two clauses, new "`a`") |
| $(\forall x)(\exists y)(Rxy \vee x \neq y)$ | `R(x,f(x)) \| -(x = f(x)).` (new "`f`") |
| $(\forall x)(\forall y)(\exists z)(Rxyz \,\&\, Rzyx)$ | `R(x,y,f(x,y)). R(f(x,y),x,y).` (new "`f`") |

- See chapters 1 and 10 of Kalman 2001 McCune 1994 for further details.

# Implementation in PROVER9: II

- PROVER9 implements many rules of inference and strategies. For our purposes, it will suffice to discuss just one of these.
- *Hyperresolution* is a generalization of disjunctive syllogism in classical logic. Here are some examples:

| | | |
|---|---|---|
| `-P \| M.` | `-P(x) \| M(x).` | `-L(x,f(b)) \| L(x,f(a)).` |
| `P.` | `P(x).` | `L(y,f(y)).` |
| ∴ `M.` | ∴ `M(x).` | ∴ `L(b,f(a)).` |

- In the third example: $x \mapsto b$, $y \mapsto b$.

# Implementation in Prover9: III

Here's a simple Prover9 proof of the validity of the following argument:

$\forall x(Greek(x) \rightarrow Person(x))$.

$\forall x(Person(x) \rightarrow Mortal(x))$.

$Greek(socrates)$.

———

$Mortal(socrates)$

```
1 [ ] -Greek(x) | Person(x)
2 [ ] -Person(x) | Mortal(x)
3 [ ] Greek(socrates)
4 [ ] -Mortal(socrates)
5 [hyper,3,1] Person(socrates)
6 [hyper,5,2] Mortal(socrates)
7 [hyper,6,4] F
```

# Implementation in Prover9: IV

- Second-order object theory must be represented in Prover9's first-order language with at least two *sorts*: `Property` and `Object`.
- E.g., one-place exemplification $Fx$ and encoding $xF$ (two forms of predication) can be represented and typed in Prover9 as follows:
  - `all F x (Ex1(F,x) -> Property(F) & Object(x)).`
  - `all F x (Enc(x,F) -> Property(F) & Object(x)).`
- Two-place predication requires a new relation: `Ex2(R,x,y)`, etc.
- Modal (S5) claims can be translated into Prover9 Kripke-style, with the use of a third sort: `Point` (*not* `World`!).
  - ```
    all F x w (Ex1(F,x,w) ->
                       Property(F) & Object(x) & Point(w)).
    ```
    ```

## Implementation in Prover9: V

- Propositions can't be defined as 0-place relations (Prover9 has no such), so a fourth sort of term is required: `Proposition`.
- With sorted terms, Prover9 requires explicit typing conditions:
  - ```
    all x (Property(x) -> -Object(x)).
    all x (Property(x) -> -Proposition(x)).
    all x (Property(x) -> -Point(x)).
    ```
- Complex properties (i.e., $\lambda$-expressions) can be represented in Prover9 using functors. E.g., we represent the property *being such that p* ('$[\lambda y\ p]$') using a functor `VAC`:
  - ```
    all p (Proposition(p) <-> Property(VAC(p))).
    ```
  - ```
    all x p w ((Object(x) & Proposition(p) & Point(w)) ->
                      (Ex1(VAC(p),x,w) <-> True(p,w))).
    ```
    ```

# Premises for Theorem: All Worlds are Maximal

- Negations of propositions are propositions. (Logical Axiom)

```
all p (Proposition(p) -> Proposition(~p)).
```

This clausifies to:

```
-Proposition(x) | Proposition(~x)
```

- 'Truth at a point' is coherent. (Logical Axiom)

```
all w all p ((Point(w) & Proposition(p)) ->
  (True(~p,w) <-> -True(p,w))).
```

This clausifies to:

```
-Point(x) | -Proposition(y) | True(~y,x)  | True(y,x).
-Point(x) | -Proposition(y) | -True(~y,x) | -True(y,x).
```

# Premises for Theorem: All Worlds are Maximal

- $Maximal(x) =_{df} \forall p(x \models p \ \vee \ x \models \neg p)$ (Definition)

```
all x (Object(x) -> (Maximal(x) <->
  (all p (Proposition(p) ->
    TrueIn(p,x) | TrueIn(~p,x)))))).
```

This clausifies to:

```
-Object(x)| -Maximal(x).
-Object(x)| -Maximal(x) | -Proposition(z)| TrueIn(z,x)| TrueIn(z,x).
-Object(x)| Maximal(x) | Proposition(f1(x)).
-Object(x)| Maximal(x) | -TrueIn(f1(x),x).
-Object(x)| Maximal(x) | -TrueIn( f1(x),x).
```

# Premises for Theorem: All Worlds are Maximal

- $World(x) =_{df} \Diamond\forall p(x \models p \equiv p)$ (Definition)

```
all x (Object(x) -> (World(x) <->
  (exists y (Point(y) &
    (all p (Proposition(p) ->
      (TrueIn(p,x) <-> True(p,y))))))))).
```

This clausifies to:

```
-Object(x) | -World(x).

-Object(x) | -World(x) | Point(f2(x)).

-Object(x) | -World(x) | -Proposition(u) | TrueIn(u,x) | -True(u,f2(x)).

-Object(x) | -World(x) | -Proposition(u) | TrueIn(u,x) | -True(u,f2(x)).

-Object(x) | World(x) | -Point(y) | Proposition(f3(x,y)).

-Object(x) | World(x) | -Point(y) | TrueIn(f3(x,y),x) | True(f3(x,y),y).

-Object(x) | World(x) | -Point(y) | -TrueIn(f3(x,y),x) | -True(f3(x,y),y).
```

## Premises for Theorem: All Worlds are Maximal

- Worlds are objects. (Proper Axiom)

```
all x (World(x) -> Object(x)).
```

This clausifies to:

```
-World(x) | Object(x)
```

# A Prover9 Proof That Every World Is Maximal

```
1 World(c1).
2 -Maximal1(c1).
4 -World(x) | Object(x).
6 -Object(x) | Maximal1(x) | Proposition(f1(x)).
7 -Object(x) | -World(x) | Point(f2(x)).
8 -Object(x) | Maximal1(x) | -TrueIn(f1(x),x).
9 -Object(x) | Maximal1(x) | -TrueIn(~f1(x),x).
14 -Object(x) | -World(x) | -Proposition(y) | TrueIn(y,x) | -True(y,f2(x)).
18 -Object(c1) | Point(f2(c1)).  [resolve (7 b 1 a)]
22 -Object(c1) | -Proposition(x) | TrueIn(x,c1) | -True(x,f2(c1)).
   [resolve (14 b 1 a)]
30 -Object(c1) | Proposition(f1(c1)).  [resolve (6 b 2 a)]
31 -Object(c1) | -TrueIn(f1(c1),c1).  [resolve (8 b 2 a)]
32 -Object(c1) | -TrueIn(~f1(c1),c1).  [resolve (9 b 2 a)]
```

```
37 -Proposition(x) | Proposition(~x).
38 -Point(x) | -Proposition(y) | True(~y,x) | True(y,x).
39 Object(c1).  [resolve (4 a 1 a)]
40 Point(f2(c1)).  [copy 18 unit_del (a 39)]
44 -Proposition(x) | TrueIn(x,c1) | -True(x,f2(c1)).
   [copy 22 unit_del (a 39)]
52 Proposition(f1(c1)).  [copy 30 unit_del (a 39)]
53 -TrueIn(f1(c1),c1).  [copy 31 unit_del (a 39)]
54 -TrueIn(~f1(c1),c1).  [copy 32 unit_del (a 39)]
60 -Proposition(x) | True(~x,f2(c1)) | True(x,f2(c1)).
   [resolve (40 a 38 a)]
63 Proposition(~f1(c1)).  [resolve (52 a 37 a)]
64 -True(f1(c1),f2(c1)).  [ur (44 a 52 a b 53 a)]
68 -True(~f1(c1),f2(c1)).  [ur (44 a 63 a b 54 a)]
70 F. [resolve (60 a 52 a) unit_del (a 68) unit_del (b 64)]
```

## Our Results Thus Far in Computational Metaphysics

- Prover9 has found proofs of all the theorems but one in Pelletier and Zalta 2000 ("How to Say Goodbye to the Third Man"), the exception involving an error of reasoning by the authors! Mace (model-building program) showed a countermodel.
- Prover9 has found proofs of all the theorems in Zalta 1993 ("25 Basic Theorems in Situation and World Theory")
- Prover9 has found a simplification of Anselm's ontological argument: The existence of God can be derived from a single non-logical premise. See Oppenheimer and Zalta, forthcoming:

  `http://mally.stanford.edu/Papers/ontological-computational.pdf`
- Our input files, output files, and proofs of the consistency of the premises (using MACE) are available online:

  `http://mally.stanford.edu/cm/`

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