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What semantics is and what purpose it serves

1. Three meanings of the word ‘semantics’

It is with certain embarrassment to note that the word ‘semantics’ is used by logicians in an ambiguous way, giving rise to various misunderstandings; this ambiguity additionally overlaps with its misleading polynomial character.

Regarding one of its meanings – the word ‘semantics’ means more or less (unfortunately, the above provision is indispensable here) something of ‘a theory of relations occurring between the language of the given theory and the domain of this theory’. Sometimes ‘semantics’, in this meaning, is applied to the whole set of such theories. In its second meaning, the word ‘semantics’, and more precisely – ‘semantics of theory T ’ – means something of ‘the domain of theory T ’. Sometimes, in this case, we say about a model of theory T . While looking at its third meaning, the expression ‘semantics’, or ‘semantics of theory T ’, respectively, means something of ‘the theory of the domain of theory T ’. Instead of the domain of theory T and the theory of the domain of theory T we can speak about ontology of theory T ; the expression ‘ontology of theory’ has thus more than one meaning, as well.

I wish to clearly make precise these three meanings of the word ‘semantics’. For comfort, I will be using the following three terms in order to do so: ‘semantics-as-theory’, ‘semantics-as-a-model-of-theory’, and ‘semantics-as-a-theory-of-model-of-theory’. The term ‘semantics’ in its first meaning does not require complementing; the term ‘semantics’ in the other two meanings is used in contexts of the following type: ‘ x is semantics of y ’.

2. Semantics-as-theory

2.1. Semantic relations

The question what semantics is in the sense of semantics-as-theory is usually answered – on the ground of logic – in the following manner: it is ‘one of the divisions of semiotics, describing relations between signs and the reality to which the signs refer, e.g. the relation designation [naming], denotation,¹ connotation, truthfulness, etc.’ (Marciszewski 1988d: 174). In this context, it also said about expressing, stating and describing. Other terms considered to be semantic include: ‘symbolizing’, ‘referring to’ and ‘relating to’, as well as “defining (a set of objects by a formula containing a variable)” (Nowaczyk 1987: 640).² A semantic relation can also be satisfaction (resp. non-satisfaction), i.e. a relation that holds between an open sentence and such objects that

¹ It is interesting to realize that there is no term ‘rangeness’.

² In A. Tarski’s work “The semantic conception of truth and the foundations of semantics” of 1944 we can read: “We say that the sentential function defines the given object if it is the only object that satisfies this function” (1944: 255).

substituting their names in the place of free variables that occur in the function, transforms this sentence into a (closed) true sentence (in the case of non-satisfaction – into a false sentence). On the other hand, representing constants by variables and – in consequence – also substituting, are not a semantic relation, or – in the second case – a semantic operation, but syntactic ones. Let us remind here at once that pragmatics is regarded as “a division of semiotics concerned with relations between a language and those who make use of it. Communicating, expressing, understanding, and the like, are examples of such a relation” (Marciszewski 1988c: 153). It is also said here that, for example, “a language expression [...] expresses [...] certain experience (a thought, a wish, a feeling, etc.) of the person” who utters this expression IN EARNEST (Marciszewski 1988e).

Designating is determined further as “a relation of the name to the designate”, providing that “the name designates each of its designates, that is each object about which this name can – applying the given meaning – be stated in compliance with the truth” (Marciszewski 1988b). It is said about naming that it is “a semantic relation holding between an individual name” and the individual whose name “the name is” (Marciszewski 1988a). By ‘denotation’ we generally understand the relation between the name and the class of its designata. Connoting, in turn, is “the relation holding between the name and a certain set of features which characterize the designata of this name” (Marciszewski 1988). Such a sentence that really acknowledges the state of affairs that occurs can be considered true; truthfulness is, thus, a relation derivative of the semantic relation of the statement, one that holds between the sentence and a certain state of affairs.

This gives the impression that between expressions and the objects subjected to them there occur VERY DIFFERENT semantic relations. In order to comment on the impression, we will distinguish two types of differences between the relations, one of which will be called ‘material differences’, and the others – ‘formal differences’.

Let us first consider the relations described in sentences of the following type:

- (1) Frederick Chopin dropped a violet.
- (2) Frederick Chopin dropped a bunch of violets.
- (3) Frederick Chopin raised a violet.

I will say that in the situations described in Sentences (1) and (3) we come to deal with relations which differ from each other materially: in the situation described by Sentence (1), Frederic Chopin is performing a different activity to that described in Sentence (3). On the other hand, the relations occurring in the situations described in Sentences (1) and (2) differ from each other formally: qualitatively the same activity is directed towards different TYPES of the object, viz. towards one violet in the first one, and a bunch of violets in the other case.

Relations regarded as semantic ones do not differ from one another materially: everywhere does it involve a certain CONVENTIONAL CORRELATION (that is STIPULATED ASSIGNING)³. There are, however, formal differences between them and they are of two kinds.

Firstly, semantic relations differ by the QUALITY of arguments, that is the type of objects between which they occur. We say:

(4) The name ‘composer’ designates (among others) Frederick Chopin.

(5) The name ‘composer’ denotes a class of composers.

Let us note that no expression ever simultaneously both designates and denotes the same object. If we expected from some expression to perform such a dual function towards the same object, we would unavoidably become entangled in paradoxes.

Secondly, semantic relations differ by the NUMBER of arguments. We will say further:

(6) The name ‘composer’ points (among others) to Frederick Chopin.

(7) The name ‘composer’ ascribes to Frederick Chopin the skill of creating musical compositions.

Sentence (6) explores a two-argument relation (an x points to a y), and sentence (7) – a three-argument one (an x attributes a z to an y).

Now, I would like to take a closer look at the above-discussed relations.

2.2. Designation

‘Designation’ is most often defined in two ways:

(1) Name ‘ N ’ designates object $X \leftrightarrow_{df} X$ is an N .

(2) Name ‘ N ’ designates object $X \leftrightarrow_{df}$ sentence ‘This is an N ’ uttered together with a gesture of pointing towards object X , is true.

It seems that the definition in the form of:

(3) Name ‘ N ’ designates object $X \leftrightarrow_{df}$ name ‘ N ’ is predicated with truth about object X .

is merely another stylization of formula (2).⁴

Formulas (1) and (2) differ at two essential points.

Firstly, object X designated by name ‘ N ’ is identified in both formulas in a different way: in Formula (1) – by means of proper name ‘ X ’, whereas in Formula (2) – with the use of the indexical word ‘this’, complemented with a suitable gesture of indicating.

³ I am developing, in this place, the thoughts contained in A. Brożek’s work under the title “Korelaty ontyczne pytań, czyli z ontologicznych podstaw semantyki” [“The ontic correlates of questions, i.e. of the ontological foundations of semantics”] (2009: 43-44).

⁴ Another stylization of Formula (1) is the definition of ‘marking’ in the categories of satisfaction, which comes from A. Tarski. We can read in his book *Pojęcie prawdy w językach nauk dedukcyjnych* [*The Notion of Truth in Languages of Deductive Sciences*] of 1933 that: “To say that name x designates the given object a , is the same as to state that object a (eventually every sequence whose proper expression is a) satisfies the sentential function of certain determined type; in the natural language it means sentential functions consisting of three successive parts: the variable, the word ‘is’ and the given name ‘ x ’” (1933: 68). It is interesting that later on, in the work “The semantic conception of truth and the foundations of semantics”, of 1944, a function of another form is given: “The given term designates the given object if the object satisfies the sentential function ‘ x is identical with T ’, where ‘ T ’ replaces the given term” (1944: 255).

In the first case, we have to know which object is assigned to name 'N'. There is well-known – although probably insurmountable – difficulty in identification via ostension connected with the other case.

Secondly, the word 'is' of Formula (1) has got a slightly different sense from the 'is' in Formula (2); the fact that in Formula (1) the argument on the right of 'is' [in inflectional languages, like Polish] has the grammatical case of the ablative, whereas in Formula (2) – the nominative, testifies to this.

It does not seem probable that the other difference as regards the language is not supported by any difference «in the world»; let us put this issue aside, though. What is vital is the fact that if Definitions (1) and (2) are to be «operational», we need to know when we have the right to say that X is an N and (with the right gesture) that this is an N – and when we do not have such a right.

All in all – the word 'is', as an ambiguous expression, requires a more precise treatment. The latter is carried out in the language of theory of classes in the following way:

(4) a is an $N \leftrightarrow_{df} a$ belongs to class of Ns .

There appears the problem of how to assign the class of Ns . Obviously, one can not say:

(5) The class of $Ns =_{df}$ class of objects which are designated by name 'N'. This could – on the ground of Formula (1) – result in a definitional vicious circle. Let us try to find a way out of this by identifying the class of Ns with denotation (in other words: extension) of name 'N'.

2.3. Denotation and connotation

If the above-indicated way is not to lead us astray again to enter a definitional vicious circle, we must not simply say:

(1) The denotation of name 'N' $=_{df}$ the class of designata of name 'N'.

It is then either necessary to list all the elements of denotation or give a hint how it is possible in each case to settle whether or not the given object is an element of denotation.

In the first case, Formula (1) of § 2.2 would take on the following form:

(2) Name 'N' designates object $X \leftrightarrow_{df}$ object X belongs to class $\{X, Y, \dots, \text{etc.}\}$.

In the other case, we are abandoning the terrain of – as one can say – referential semantics and refer to the notion of CONNOTATION.

(3) Property Pr is connotation of name 'N' \rightarrow (name 'N' designates object $X \leftrightarrow_{df}$ object X has property Pr).

But, as a matter of fact, what is it – the connotation (in other words: content) of a name? One has to reject here in advance an explanation in the form:

(4) Property Pr is the connotation of name 'N' \leftrightarrow_{df} property Pr belongs to all and only designates of name 'N'.

Let us neglect (for the reason of simplifying) the fact that certain names – with certain assumptions – do not have designates at all, and despite this seem to

have connotation, as well as the case where a given name can possess many connotations determined in such a way. The thing is that (4) in connection with (3) yields a strange formula:

(5) Property *Pr* belongs to all and only designates of name ‘*N*’ \rightarrow (name ‘*N*’ designates object *X* \leftrightarrow_{df} object *X* has property *Pr*).

The way out of the problem – if we are not able to meet the challenge of constructing Formula (2), that is to list all the designates of name ‘*N*’ – I can see in abandoning the view that Formula (1) of § 2.2 is the definition of ‘designating’.

2.4. Designation iterum

I propose to define ‘designation’ through making reference to the action of indicating:⁵

Object *X* is a designatum of name ‘*N*’ \leftrightarrow_{df} person *Pe* uses name ‘*N*’ to indicate object *X*.

The following dependence holds:

Object *X* is a designatum of name ‘*N*’ \rightarrow (person *Pe* utters with conviction the sentence ‘*N* is a *P*’ (resp. ‘*N* is *P*-like’) \rightarrow person *Pe* uses name ‘*N*’ to indicate object *X*).

This proposition requires a few comments which – as a matter of fact – refer also to the formulas proposed below.

Firstly, Formulas (1) and (2) do not include *explicite* quantifiers, yet *implicite* are quantified by generalizers (or by general quantifiers).

Secondly, in Formula (1), the phrase ‘(each) person *Pe*’ is a far-fetched idealization that ignores the fact that *de facto* not all people use all names of the given language, that some people sometimes use the language in a wrong way or consciously do it in a way that differs from what it «should be», etc. For this reason the following formula would seem closer to the reality:

Object *X* is a designate of name ‘*N*’ \leftrightarrow_{df} name ‘*N*’ serves to indicate object *X*.

The phrase ‘serves to’ would then mean more than simply being-used-for. We will say that the name ‘cow’ serves to indicate cows, even if someone uses the name to indicate some hippopotamus.

The definiens of Definition (1) has the form ‘person *Pe* uses name ‘*N*’ to indicate object *X*’, among others, in order to facilitate perceiving a relation between Formulas (1) and (2).

Thirdly, Formula (1) is consciously «pragmatized». Indeed: we know what the designate of the given name is, what objects are generally indicated by these names. This is so, at least, on the ground of the natural language.

⁵ In the proposed definition of ‘designation’ and ‘connotation’, I follow the solutions accepted by A. Brożek in the work “Kwadrat, zys i Tryglaw, czyli o Jacka Jadackiego koncepcji funkcji semnacyjnych nazw” [“The square, the zys and the tryglaw, that is about Jacek Jadacki’s conception of the semantic functions of names”] (2007).

2.5. Denotation and connotation iterum

We will leave the definition of ‘denotation’ of § 2.3 unchanged, but – at the same time – devoid of the threat of the vicious circle:

Denotation of name ‘ N ’ \leftrightarrow_{df} class of designates of name ‘ N ’.

Let us stress that this formula can be modified in a variety of ways in dependence on whether for the purpose of denotation of name N we want to include all the objects that name N serves to indicate (thus also those that have never been and will not be indicated effectively by means of name N), or only some of them (e.g. those that somebody has already indicated by means of name N). In turn, I would define ‘connotation’ through reference to actions of assigning a certain property to an object.⁶

Object X is a designatum of name ‘ N ’⁷ \rightarrow (property Pr is the connotation of name ‘ N ’ \leftrightarrow_{df} person Pe uses name ‘ N ’ to assign property Pr to object X).

It looks then that ‘indicating-something’ and ‘assigning-something-to-something’ are primitive terms which refer to the fundamental mental operations. The following dependence holds then:

(3) (Object X is a designate of name ‘ N ’ \wedge property Pr is the connotation of name ‘ N ’) \rightarrow (person Pe utters with conviction the sentence ‘ X is an N ’ (resp. ‘ X is N -like’) \rightarrow person Pe uses name ‘ N ’ to assign property Pr to object X).

Let us pay attention – not going into details – to the fact that on the basis of Formula (3) we have the right to accept the following dependence similar to that mentioned in Formula (1) of § 2.2:

(4) Object X has property Pr \rightarrow [(Name ‘ N ’ designates object X \wedge property Pr is the connotation of name ‘ N ’) \leftrightarrow X is an N].

On this basis we can acknowledge that:

(5) (Property Pr is the connotation of name ‘ N ’ \wedge X is an N) \rightarrow X has property Pr .

2.6. Types of names due to designation and connotation

After such terminological establishments it is relatively easy to describe a linguistic phenomenon which – until now – has been the source of misunderstandings and debates.

If it comes to designation and connotation, then – as it is well-known – the following combinations are possible:

- (a) the given name both designates and connotes something;
- (b) the given name designates something but it does not connote anything;

⁶ As in the case of many terms drawn from the natural language, here – in the same way – one should remember that on the ground of this language the phrase ‘ A assigns C to B ’ is used only when C does not belong to B in reality. There is no such restriction in our formula.

⁷ In formulas (2)-(4) of this section, the condition/factor “object X is the designatum of name ‘ N ’” is added because if the given name has connotation, then an appropriate property is assigned to all and solely its designata. If, for instance, formula (2) contained only “property Pr is the connotation of name ‘ N ’ \leftrightarrow_{df} person Pe uses name ‘ N ’ to assign property Pr to object X ”, it would not be clear what kind of quantifier were to bind variable ‘ X ’.

(c) the given name does not designate anything, but it connotes something;

(d) the given name neither designates nor connotes anything;

Names of type (a) can be identified with non-empty general names, those of type (b) – with non-empty individual names (that is with proper names); names of type (c) – with empty general names, and those of type (d) – with empty individual ones. Let us see, as an example, how the proposed definitions function for non-empty individual names.

(1') I am the designate of the name 'Jacek Jadacki', since people who know me⁸ use this name to indicate my person.

(2') Since I am the designate of the name 'Jacek Jadacki', if somebody utters with conviction the sentence: 'Jacek Jadacki is – let us assume – a philosopher', then he uses the name 'Jacek Jadacki' to indicate my own person.

(3') Denotation of the name 'Jacek Jadacki' is a singleton, I am the sole element of which.

(4') Since nobody uses⁹ the name 'Jacek Jadacki' to assign some property to me, the name has no connotation.

(5'/6') Nobody (in view of the above) utters, with conviction, the sentence in the following form 'The designate of the name "Jacek Jadacki" is Jacek Jadacki'.

2.7. Truthfulness and ascertaining

Truthfulness is not, obviously, as it is sometimes said for short, a semantic relation, but a property of sentences which belongs to them due to the fact that they remain in a determined semantic relation with something. Let us remind here that a sentence that ascertains the really occurring state of affairs is considered true.¹⁰ Can't we then reduce ascertaining to designation or connotation?

In my opinion – nothing stands in the way to do so. As a result of this reduction we would treat sentences as certain particular names, that is names of states of affairs. We would then have to acknowledge that:

(1) Sentence '*p*' designates the same state of affairs as the name 'so that *p*'.

⁸ As regards the need for the insert 'who know me' – see comments with reference to § 2.4.

⁹ Here, one should again remember about the comments referring to § 2.4. They need adding one more observation that the name '(Jacek) Jadacki', in some circumstances, CAN be used in this way and that one can say that it connotes something; compare, for example, the context 'He is another Jadacki'.

¹⁰ Some, perhaps, could say that a true sentence ascertains the OCCURRENCE of a real state of things. I would see it in the following way. The sentence: "Sentence *S* ascertains the occurrence of the real state of things *ST*", or it says the same as the sentence: "Sentence *S* ascertains the really occurring state of things *ST*", or it says something different. In the second case, sentence *S* would ascertain in particular not (the really occurring) state of things *ST*, but some other state, that is that the state of things *ST* is the really occurring state of things. Hence, the fact that Frederick Chopin was a prepossessing man is ascertained by the true sentence, "Frederick Chopin was a prepossessing man" since this state of things really occurs; on the other hand, the fact that it really occurred that Frederick Chopin was a prepossessing man is ascertained by the true sentence, "That Frederick Chopin was a prepossessing man, occurs in reality". It is worth adding that the word 'occurs' means the same as 'exists', the first having only a slightly different grammatical connectability from the other.

There are reasons to believe that Frederick Chopin was in love (of course, in a period of his life) with Maria Wodzińska; let us assume that it was so in reality. With this assumption, the name ‘that somebody loves somebody’ (and also, among others the name ‘Frederick Chopin loved Maria Wodzińska’) designates, among others, that Frederic Chopin loved Maria Wodzińska; the same state of affairs is also designated by the sentence, ‘Somebody loves somebody’ (as well as, among others, by the sentence, ‘Frederick Chopin loved Maria Wodzińska’). The fact (that is the really occurring state of affairs) that Frederick Chopin loved Maria Wodzińska causes the names ‘that somebody loves somebody’ and ‘that Frederick Chopin loved Maria Wodzińska’ to be non-empty, and the sentences, ‘Somebody loves somebody’ and ‘Frederick Chopin loved Maria Wodzińska’ to be true. The sentence, ‘Frederick Chopin loved Jane Stirling’ is false and the corresponding name is empty, since that Frederick Chopin loved Jane Stirling is not a fact.

Formula (1) is not to be sustained if ‘designating’ is defined in the way as in Formula (1) of § 2.2. After all, we can not say:

(2) Sentence ‘*S*’ designates object $X \leftrightarrow_{df} X$ is an *S*.

If ‘*S*’ is a sentence, then the right side of the equivalence is ungrammatical.

On the other hand, the appropriately adapted formulas of § 2.4 maintain their validity:

(3) The state of affairs X is a designate of sentence ‘*S*’ \equiv_{df} person P uses sentence ‘*S*’ to indicate state of affairs X .

(4) The state of affairs X is a designate of name ‘*S*’ \rightarrow (person P utters with conviction sentence ‘*S*’ \rightarrow person P uses sentence ‘*S*’ to indicate state of affairs X).

(5) Denotation of sentence ‘*S*’ $=_{df}$ class of designates of sentence ‘*S*’.

Obviously, there arises the question: What would be the connotation of a sentence in this framework? One of the answers – assuming certain characteristics of states of affairs – would be that sentences do not connote anything.

Some – including myself in the past – accept that a special feature of sentences is that they are suitable to express convictions (resp. their content). In this light, as it was discussed above, expressing convictions is nothing else but indicating them. Sentence ‘ p ’ would perform then two functions: it would indicate that p , but also that somebody that utters ‘ p ’, is convinced that p ; in the other case it would be a synonym of the sentence, ‘I am convinced that p ’. If somebody took a dislike to this manner of speaking, it could be accepted that it is not the very sentence ‘ p ’ itself which indicates the content of the appropriate conviction, but the UTTERANCE of sentence ‘ p ’.

Let us add that the same can be said about names: the name ‘Frederick Chopin’, on the one hand, indicates Frederick Chopin, on the other one – the name (or its usage) indicates a representation – or in a more general sense – presentation of its only designate (possessed by the user of this name).

2.8. Satisfaction

In the way that does not raise any objections, it is possible to define ‘satisfaction’ for open sentences, that is containing at least one variable. Let us consider this, using the simplest example.

(1) Sequence of objects $\langle a, b \rangle$ satisfies the open sentence ‘ x loves y ’ $\leftrightarrow_{df} a$ loves b .

It can happen that an open sentence of the type ‘ Rxy ’ is satisfied by a certain sequence of objects or by all sequences of objects of a certain set. We then have respectively:

(2) A certain sequence of objects satisfies the open sentence ‘ Rxy ’ $\leftrightarrow_{df} \exists x \exists y (Rxy)$.

(3) Each sequence of objects satisfies the open sentence ‘ Rxy ’ $\leftrightarrow_{df} \forall x \forall y (Rxy)$.

The last one occurs, for instance, when it comes to an equality relation in the class of natural numbers and the sentence has the form ‘ x equals x ’.

How do the open sentences ‘ x loves y ’ and ‘ x equals x ’ relate to the closed sentences ‘ a loves b ’ and ‘ a equals a ’? The latter is a result of substitution for free variables in the former – of appropriate constants of a given set. Formula (1) can be then given the following «operational» form:

(4) The sequence of objects $\langle a, b \rangle$ satisfies the open sentence ‘ x loves y ’ \leftrightarrow_{df} upon substituting constants ‘ a ’ and ‘ b ’ for the free variables ‘ x ’ and ‘ y ’, respectively, we obtain a true sentence.

For instance, the open sentence ‘ x loves y ’ is satisfied by the pair \langle Frederick Chopin, Maria Wodzińska \rangle , since Frederic Chopin did love Maria Wodzińska. The pair \langle Frederick Chopin, Maria Wodzińska \rangle , whose elements belong to the values of the variables of our open sentence, satisfies this sentence because it becomes true when we accept the first element of the pair as the value of variable x , and the other – as the value of variable y , and when (the fact is that) Frederick Chopin loved Maria Wodzińska. The open sentence ‘ x loves y ’ is also – now on the power of the convention – satisfied, among others, by every sequence beginning with Frederic Chopin and Maria Wodzińska; the remaining places of such sequences are (as it is said) invalid.

In turn, the open sentence ‘ x is identical with x ’ is satisfied by each sequence of objects since whatever we substitute for the variables in this sentence (and more precisely for variable ‘ x ’) we will obtain a true sentence out of it.

It is often heard that truthfulness – and thus a feature of closed sentences which do not possess free variables – can be reduced, in terms of the definition, to satisfiability. In particular, it is said, for example (Quine 1970: 60):

For a sentence devoid of free variables no element of the sequence is valid. A closed sentence is thus either satisfied by each sequence or is not satisfied by any, in dependence simply on whether it is true or false.

And thus (Quine 1970: 60-61):

Each sequence satisfies each true sentence, and none does a false sentence. The definition of *truth* in terms of satisfiability is indeed very simple: SATISFACTION BY ALL SEQUENCES.¹¹

Certainly, this is not in accordance with the above-presented intuitions related to satisfaction. After all, in order to check whether some objects satisfy the given sentence we need to substitute for the free variables in the sentence some constants which designate the objects; something like that, however, can not be done in an effective way, since – in a closed sentence *ex definitione* – there are no free variables. We can, obviously, accept – by convention – that a true sentence is merely something of a sentence satisfied by all sequences of objects of a certain set – yet the «operational» value of such a convention is doubtful. With this convention both the sentence: ‘Frederick Chopin loved Maria Wodzińska’ and the sentence: ‘Frederick Chopin died in Paris’ would be satisfied by the same sequences of objects, that is by all such sequences. This

¹¹ Let us go back to the source of these and similar formulations, that is to A. Tarski’s works. In the book *The Notion of Truth in Languages of Deductive Sciences* of 1933 we read: “IT IS EASY [my own underlining, JJ] to realize that whether the given sequence satisfies the given sentential function depends exclusively on these words of sequences which correspond to [...] free variables of the function. Thus, in the extreme case, when the considered function is a sentence, and therefore does not contain free variables at all [...], satisfaction of the function by the sequence does not in any way depend on the property of the words of the sequence. Then there are only two possibilities left: either each infinite sequence of classes satisfies the sentence or no such sequence satisfies it. Sentences of the first type [...] are just TRUE SENTENCES; sentences of the other type [...] can be called FALSE SENTENCES, respectively” (1933: 69). NB. It is said here about a sequence of CLASSES, since the analyzed sentences belong to the theory (*scil.* algebra) of classes, and classes understood in this way are values of free variables. Further – in the references – we read: “In the whole of the above construction one could use finite sequences with a varying number of words instead of infinite sequences. [...] Modification of the construction would consist in that all «redundant» words, words that do not exert any influence on the relation of satisfaction would be removed from sequences satisfying the given sentence function. [...] Assets of such a modification from the point of view of naturalness and natural intuition are obvious; nevertheless, thorough realization exposes certain defects of logical nature; the definition of [...] [‘satisfaction’] takes on a more complicated form. As regards the notion of truth, it needs to be observed that – on the ground of the above concept – the sentence, i.e. function without free variables can be satisfied by only ONE sequence, that is an «empty» one that does not possess a single word; we should then apply the word true with reference to sentences which are indeed satisfied by an «empty» sequence. Some artificiality of this definition will undoubtedly disturb all those that are not satisfactorily used to specific «tricks» used in mathematical constructions” (1933: 69-70). The suggestiveness of these formulations is so great that they are repeated – without any alterations – by many authors in contexts which rather exclude the hypothesis that the formulations are truly comprehended by these authors. I would rather admit at once that I count among those who are suspicious of the «tricks» mentioned above. In the work “The semantic conception of truth and the foundation of semantics” of 1944 we read: “When we have at last the general definition of satisfaction, let us notice that it applies AUTOMATICALLY [my own underlining, JJ] to these particular sentence functions which do not contain any free variables, i.e. sentences. IT TURNS OUT [my own underlining, JJ] that there are only two cases possible for sentences: either the sentence is satisfied by all objects or it is satisfied by no object at all. We, then, come to the definition of the truth and the falseness, saying simply that A SENTENCE IS TRUE WHEN IT IS SATISFIED BY ALL OBJECTS, AND THAT IT IS FALSE – IN THE OPPOSITE CASE (1944: 250). It is significant that in the work “Aussagenkalkül und die Topologie” of 1938, A. Tarski used the expression, “[the given sentence] holds (in other words: is valid or is satisfied) in a [given] space”, yet the sentence is a sentence of topology, and the space, which is referred to, is a topological space (1938: 190).

conclusion does not, however, equip us with any tool that allows differentiating the true sentences yet different from each other with respect to their content.

The source of the – as far as I see it erroneous – view that the truthfulness (of a closed sentence) can be reduced by definition to satisfaction is mixing up of the notion of TRUTHFULNESS OF CLOSED SENTENCES with that of – let us call it – TRUTHFULNESS OF OPEN SENTENCE. Let us take, for example, the sentence calculus, that is the theory of inter-sentence relations. The theses – that is laws – of this theory are – as it is well-known – tautologies, therefore (as it is popularly said) schemas of exclusively true sentences. Let us consider, for instance, the law of identity ' $p \rightarrow p$ '. It is satisfied by every sentence and thus it is a law of this theory. About each sentence that is substitution of the sentence calculus law, hence satisfies this law, we say that it is true logically. We can make a deal that we will say about each law (i.e. about tautology) of this calculus that it is true. The formula of sentence calculus would be true in this sense if it were satisfied by each sentence. The same can be said, for instance, about formulas of predicates calculus. For example, the formula ' $x = x$ ' is – in this sense – true as it is satisfied by each individual element; in this case this is even marked with an appropriate quantifier, giving ' $\forall x (x = x)$ '.

Now, let us look closer at the formula:

(5) Sentence (open) ' $p \rightarrow p$ ' is logically true (on the ground of the sentential calculus) \leftrightarrow_{df} each closed sentence which is the value of variable ' p ' satisfies the sentence ' $p \rightarrow p$ '.

Formula (5) can be transformed into the following formula:

(6) The open sentence ' $p \leftrightarrow p$ ' is logically true (on the ground of the sentential calculus) \leftrightarrow_{df} each closed sentence which is the value of variable ' p ' transforms the open sentence ' $p \rightarrow p$ ' into a true closed sentence.

The truthfulness that is included in the *definiensis* of Definition (6) can not be the truthfulness meant in the *definiendum* of the definition – under the threat of the definitional vicious circle. Indeed, within the sentence calculus nothing can be said about the truthfulness of *simple* closed sentences belonging to the values of sentence variables of the language of this calculus. The notion of – as we have called it – TRUTHFULNESS OF OPEN SENTENCES is not applied to closed sentences. In consequence, the notion of SATISFACTION cannot be employed here either in the sense it is used with reference to open sentences.

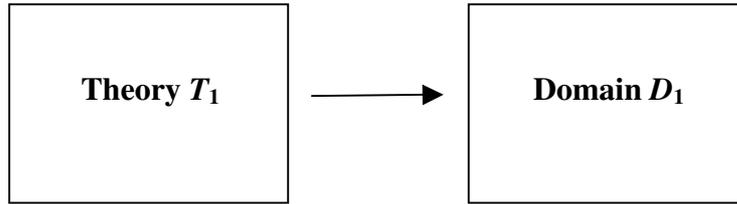
3. Semantics-as-a-model-of-theory and semantics-as-a-theory-of-model-of-theory

3.1. The domain of reality and its theory

By the theory of reality R , I understand any description of what this reality R contains. They can be both individual sentences and general ones, the latter being shortenings of the conjunction of the former. Sometimes the so-called rules are added, that is prognoses-hypotheses relating to the objects that are not

specified. A diagram and, perhaps – a matrix just as well – can be a peculiar kind of theory.

Let us assume that Theory T_1 is a theory of Domain D_1 .

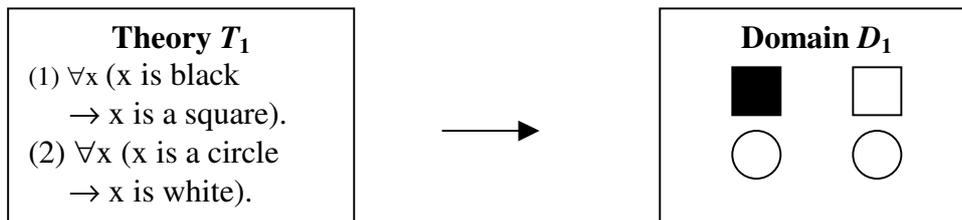


Let theory T_1 be a set of certain theorems and domain D_1 – a set of certain objects.

Now, let us assume that domain D_1 consists of four figures: two squares and two circles, still one of the squares is black and the other one – white, and both of the circles are white.

Let us assume further that language L_1 of theory T_1 (hereafter language L_1 for short) contains the following constants:

- quantifier ‘ $\forall x$ ’;
- copula ‘is’;
- connective of implication ‘ \rightarrow ’;
- names ‘white’, ‘black’, ‘circle’, and ‘square’.



Let the following two theorems be part of Theory T_1 :

- (1) $\forall x$ (x is black $\rightarrow x$ is a square).
- (2) $\forall x$ (x is a circle $\rightarrow x$ is white).

3.2. The truthfulness of theory

The Theorems (1) and (2), belonging to theory T_1 of § 3.1 are true about the objects which are part of domain D_1 . How do we know this? There are two sources of this knowledge:

(a) We know about the truthfulness of Theorems (1) and (2) of theory T_1 (hereafter about the truthfulness of theory T_1), among others, because we can see the PICTURE presenting the content of domain D_1 .

(b) We know about the truthfulness of theory T_1 , among others, because we know the DESCRIPTION of the content of domain D_1 .

The sources, however, are not sufficient to obtain knowledge that theory T_1 is true about the objects belonging to domain D_1 . One more source is still necessary:

(c) We know about the truthfulness of theory T_1 , among others, because we understand language L_1 , and – in particular – we understand quantifier ‘ $\forall x$ ’, copula ‘is’ and connective ‘ \rightarrow ’, and we know that the names of the language possess the following designates:

‘white’	\longrightarrow	  
‘black’	\longrightarrow	
‘a circle’	\longrightarrow	 
‘a square’	\longrightarrow	 

Let us suppose that we want to base our knowledge about the truthfulness of theory T_1 on (a) and (c). Domain D_1 is then identified by means of our perception, while language L_1 is understood on the basis of the ostensive identification of its reference (to simplify, let us neglect the issue of understanding quantifier ‘ $\forall x$ ’, copula ‘is’ and connective ‘ \rightarrow ’).

Let us assume, in turn, that we want to refer to sources (b) and (c). Then domain D_1 is identified verbally – through a description; let us designate the language of the description as $J(D_1)$. This description could have a form as given above or, for example, one acknowledging that domain D_1 contains four figures: a black square, a white square, a white circle and a black circle.

3.3. Semantics-as-a theory-of-model-of-theory

I would like to defend the view that semantics-as-a-theory-of-model-of-theory- T is simply another theory of semantics-as-a-model-of-theory- T , and thus the reality (or part of it) which theory T concerns. In consequence, I believe that we come to deal here with – on the one hand – certain reality (or part of it), on the other one – with two different theories of this reality (or its part). This reality here – is semantics-as-a-model-of-theory- T , and the theories are theory T and semantics-as-a-theory-of-model-of-theory- T , respectively. Thus, it can be said that these theories (or their respective fragments) are mutually equivalent, and the language of one of the theories can be considered a translation of the other one. If we assume (as we have just done above) that theory T_1 describes domain D_1 in language $L(D_1)$, we have then the description of domain D_1 done in two languages: L_1 and $L(D_1)$.

Let us notice that in a certain variation of the situation – I would call it «degenerated» – language L_1 is identical with language $L(D_1)$. We would then have in our case:

Theorem (1) [‘If something is black, it is a square’] is true, since if something is black, it is a square.

Theorem (2) [‘If something is a circle, it is white’] is true, since if something is a circle, it is white.

Let us add at once that what has been said above about theory T_1 , domain D_1 , language L_1 and language $L(D_1)$ was uttered in a language different to language L_1 and language $L(D_1)$; let us denote this language as L' . Some would say that language L' is – in this case – a metalanguage for language L_1 and language $L(D_1)$.

3.4. What semantics-as-a theory-of-model-of-theory is needed for

There is no need to dwell longer on semantics-as-a-model-of-theory. The identification of reality (or its part) which theory T_1 concerns is indispensable – as it has been shown above – to comprehend theory T_1 , as well as to establish whether it is true.

I regard the requirement sometimes posed – especially towards the newly-constructed theory T_1 – to provide its (or for it) semantics-as-a-theory-of-model (hereafter in this section: semantics_{TMT}) of theory T as one that every such theory should be at once translated by its constructor into the language of another – generally already existing – theory treated just as semantics_{TMT} of the first theory.

When is such a requirement justified?

First, when the addressee of the new theory does not understand it – therefore it needs formulating in the language known to the addressee.

Second, when it is not possible, by extralinguistic means, to indicate the reality (or part of it), which the new theory concerns. This is so, e.g. – it seems – in the case of the domain of numbers. Let us remind that without indicating the domain we will not comprehend the theory. Let us note that this is a postulate relating also to the present considerations which can be considered to be a certain proto-theory lectured in the appropriate part of the English language. If I feel released from providing semantics_{TMT} for this proto-theory it is because the language of the proto-theory is an existing language.

Third, perhaps it is easier to obtain the FEELING of understanding what somebody means when they pass it to us in more than one language.

Fourth, because many (to say cautiously) theories have more than one interpretation, acknowledging two different theories to be equivalent (that is concerning the same domain) facilitates identifying intended interpretation (or a class of such interpretations).

Fifth, when semantics_{TMT} of theory T_1 is a theory of domain D_1 , which is better (or recognized as better) for some reason important to us than the very theory T_1 itself, in particular, for example:

- (a) is a theory of domain D_1 , easier than theory T_1 ;
- (b) is a theory of domain D_1 , more comprehensible than theory T_1 ;
- (c) is a theory of domain D_1 , better adjusted than theory T_1 ;

We will make a broader comment on cases (a)-(c).

3.5. The argument of economicality

The relation of consequence is regarded as one of the most important relations considered in logic: it is even said sometimes that logic is a theory of

consequence. Can we limit ourselves to the syntactic definition of ‘consequence’? In compliance with it – sentence ‘ q ’ is a consequence of sentence ‘ p ’ when the implication ‘If p , then q ’ is a tautology. Moreover, in order to prove – in a purely formal way within the given (let us say it in this way: objective) logical theory – the tautological character of the given formula it is necessary to either acknowledge it to be an axiom or provide its proof on the ground of accepted axiomatic basis and accepted rules of demonstration. In both cases we come up against serious difficulty. In the first case the question arises: On what basis do we regard the accepted axioms as tautologies? In the other one – there may arise justified doubts whether our axiomatic basis is full for sure (i.e. if it will be possible to deduce all the tautologies out of it). Besides, it is often easier to prove that some sentence S is not a semantic consequence of a set of given sentences than to prove that it is not their formal consequence: it suffices to construct a model in which these sentences are true and where sentence S is not true. A variation of such a manner of conduct with reference to the sentence calculus is the matrix method (zero-one).

Tautology is a scheme of exclusively true sentences; examining the latter requires applying the procedure of substitution. So as not to get entangled in semantics_{TMT} we would have to have a list of true sentences at our disposal.

3.6. The argument of intuitiveness

Let us return to the issue of understanding the connective ‘ \rightarrow ’ (see § 3.2). This understanding would be made easier if a relation of the, e.g., cause-effect type, or another – if it can be said in a free way – dynamic relation, corresponded to it in domain D_1 . Yet, there are no such relations «seen» in domain D_1 .

Let us describe a situation in domain D_1 by means of language L_2 , differing from language L_1 in that instead of connective ‘ \rightarrow ’ there occur, in it, two conjunctions:

- the connective of negation ‘ \sim ’;
- the connective of conjunction ‘ \wedge ’.

Let us now take a look at theory T_2 of domain D_1 including two theorems:

- (1) $\forall x \sim (x \text{ is black} \wedge \sim x \text{ is a square})$.
- (2) $\forall x \sim (x \text{ is a circle} \wedge \sim x \text{ is white})$.

Theorem (1) states that in domain D_1 there is no such figure that would be black but would not be a square. On the other hand, Theorem (2) maintains that in domain D_1 there is no such figure that would be a circle, but would not be white.

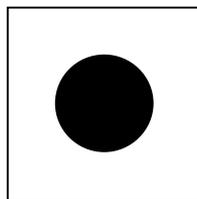
What Theorems (1) and (2) say is directly «seen» in domain D_1 and – at least it is seen in a more evident way than what Theorems (1) and (2) of § 3.1 do. This would justify the use of theory T_2 as semantics_{TMT} for theory T_1 with respect to the fact that the first is more intuitive than the other one. One needs only to agree that both theory T_1 and theory T_2 describe the same aspect of

domain D_1 . It would be relatively simple if domain D_1 had only the aspect described in these theories.

Let us note that, in practice, this does not occur, does it?

3.7. The argument of categorical adequacy

Let us look at the following diagram:



Now, let us consider the following DESCRIPTIONS of what we can see inside the diagram:

(O₁) This circle is black.

(O₂) The [colour] black is circular.

(O₃) This is both circular and black.

Are the descriptions (O₁)-(O₃) identical? The answer to this question depends on THE KIND OF RESPECT IN WHICH the identity is concerned. That they are not identical descriptions in respect of the shape of the inscriptions used is obvious to everybody. Are they, however, identical in respect of WHAT is described in them? Are they identical in respect of what they say about the content of the diagram?

Here, most probably, there will appear differences in opinions. It is sometimes thought that there are merely ontological predilections behind these differences. Some are inclined to – as it is called – «class» ontology; some to «collectivist», some – to situational, some – to eventistic, others – to reistic, and still others – to «attributivistic». The names of these ontologies – understood both as semantics-models-of-theories and semantics-theories-of-models-of-theories – signal that the following are, in succession, considered elementary components of reality (or its part) described in theories: sets, integers, states of affairs, events, things and properties.¹²

Perhaps sometimes predilections are decisive here, but *de gustibus non est disputandum*. Sometimes, however, there is something more that is concerned: adequacy towards reality (or its part) as it is given to us in perception. This can and must be discussed.¹³

¹² Let us note that in these categories one can consider also a pair of psychological theories: psychology-of-spirit and psychology-of-behaviours (that is behaviourism).

¹³ The non-adequacy of theories of classes as semantics-of-model-of-theory for the natural language (obviously, not using the terms introduced here) is drawn attention to by A. Nowaczyk in the course-book written jointly with Z. Żoźnowski, entitled *Logika i metodologia badań naukowych dla lekarzy* [*Logic and Methodology of Scientific Research for Medical Doctors*] (1974: 107), as well as by A. Brożek in the work “Reprezentacja a komunikacja” [“Representation and communication”] (2009a: 138).

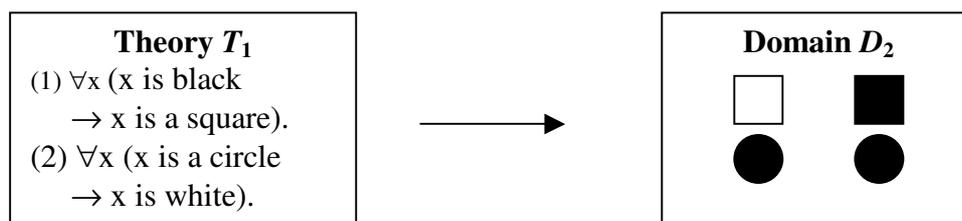
4. Semantics-as-a-theory iterum

The considerations presented in § 2 are part of semantics-as-theory (hereafter: semantics). I want to clearly stress that they ARE PART OF it, but do not CONSTITUTE it, if one understood by it a finished (and even the more so – formalized) theory. For example, it is not expected of semantics, among others, to provide for semantic functions of expressions an exhaustive list of their rules of «putting together»; there is no anything like that here.

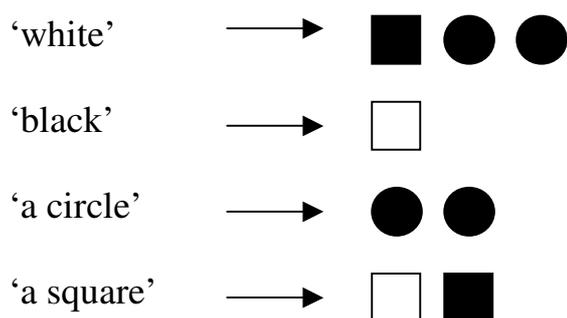
An instance of finished semantics (let us add: by assumption – referential) is the theory of models; it is, however, as it is well known, a theory relating to formalized languages and relating to other languages in as much as they will allow to be reconstructed in a formalized language.

In conclusion, I would like to share an opinion concerning how the theory of models (in a certain version) deals with interpreting the notions of NECESSITY and POSSIBILITY, regarded sometimes as very significant ontological ones.

In §§ 3.1-3.2 the situation was that for the given domain we had two different theories at our disposal. Let us notice now that language L_1 considered there can be understood in the way that theory T_1 will be a theory of domain D_2 different from domain D_1 .



It is sufficient then that the names of language L_2 were to have the following reference:



Theory T_1 would be thus – in dependence on the understanding of its predicators – a theory of both domain D_1 and domain D_2 . There can be more such POSSIBLE domains selected for theory T_1 , especially if one takes into account POSSIBLE interpretations of other constants of language L_1 .

Well, it is sometimes said that not until all POSSIBLE (*scil.* admissible) domains, in which some theory is true, are taken into consideration is understanding the functor of necessity (therefore possibility) *de dicto* possible.

Let us suppose that some theory has exactly two admissible domains. We can then, for example, say – in a simplified way – that the theorems of this theory which are true in the both domains are necessary theorems and the true ones only in one of them – (let us say) accidental theorems.

Obviously, the key issue here is the question in what way it can be established whether or not the given domain CAN BE a domain of the given theory.

I have the impression that this cannot be done in a satisfying manner – unless we identify ‘necessity’ with ‘reality’ (with what vitally takes place in the domain that we are examining), and invest ‘possibility’ with the epistemic sense.

Then, however, the both notions – necessity and possibility – can be removed from our language for its (let us call it this way) semantic power.

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