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Changeability and Constancy in Frame of Consistency. Modal Logic LC□

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1. Introduction

Any formal system since its formulation lives with its own life and it may happen that applications of it admit surprising interpretations which determine even more surprising theories. However in case of many logics there are underlying some explicit motivations which restrict the range of possible interpretations to these which are more plausible. The presented analysis is concentrated on such plausible interpretation of LC logic of change which was originally motivated by Aristotelian idea of substantial changes and now will be referred to some fragments of Leibnizian theory of change.[1]

The common way of analysis of changeability usually links it with a passage of time and so it is often described using temporal notions. In frame of philosophical logic such approaches have been already undertaken. Appropriate definitional extensions of the von Wright's calculus And Next and other temporal calculi have been discussed e.g. in (Wajszczyk, 1995).[2] In these considerations we take a converse direction and we treat the notion of change as a primitive one, which may also be used to define some temporal operators. This intention is already expressed in the logic of change LC formulated in (Świętorzecka, 2008; Świętorzecka, Czermak, 2012).

As it was mentioned, the system LC had as its philosophical background the Aristotelian theory of substantial changes. It was designed as a formal basis of the description of *becoming* and *disappearing* (*individual*) *substances*. Now we refer also to the Leibnizian philosophy of time and change with the crucial assumptions that: (i) change is not a cause of a contradiction, (ii) change is prior to time and (ii) time and change are of relational nature i.e. their description should be grounded on some specific relations between individuals. The sketched

[1] We present only a sketch of our approach extensively discussed in (Czermak, Świętorzecka 2013).
[2] Besides descriptions of change in frame of systems of von Wright, Prior and Clifford, Wajszczyk proposed also two own temporal logics of dichotomic and continuous changes.

leibnizian interpretation in (2) is mentioned to justify both axiomatization and semantics of LC, which are presented in (3). In the last part (4) we enrich our description by operator □ understood as a counterpart of unchangeability (constancy).

2. A philosophically plausible preformal interpretation of LC logic of change

Let us only remind of the main features of substantial changes considered by Aristotle. According to Aristotle all types of changes do not trigger inconsistency, they can be described in frame of theories based on classical logic. So called *substantial changes* have two components: disappearing (destruction) and coming into being (generation) of substances. Substantial changes concern transition from existence of a substance to the existence of the next one – they refer to the relation of *enability* between substances. It holds when an existence of one substance *enables* the existence of another one. They are dichotomic, do not yield contradictions and they are prior to time. (for a detailed description c.f. Świętorzecka 2008).

Leibnizian ontology is different from Aristotelian approach in many respects, however in case of philosophy of time and change Leibniz assimilated essential Aristotelian ideas. In (NE) Leibniz quotes several times the Aristotelian description of change which is understood as a transition from *potential* to *actual existence*. Leibniz follows Aristotle in the conviction that time is dependent of change and moreover it is only *the aspect of movement* (Phys, IV, 11, 219b-3). According to Leibniz *time is the measure of change* (NE, II, 152 ff) and even *does not enter into the definition of change* (VE,168). Similarly to Aristotle, Leibniz also considers as subjects of changes atoms of *inherence* relation: *individual substances*. After all he understood them in a different way to Aristotle: Leibnizian substances are called *monads* which are *eternal* and *imperishable* and so they can not generate time series.[3] Actually Leibniz links changeability in general with the transition of possessing/non possessing by monads some *attributes* and a collection of attributes which allows to identify some monad is considered as (elementary) *state* of this monad. In frame of Leibnizian considerations changes are transitions concerns *stages* of a monad, that all are already *comprised* in its *notion*[4]:

> *it is the nature of created substance to change continually in accordance with a certain order, which conduct it spontaneously (if one may use the word) through all its stages, in such a way that someone who saw everything would see in its present state all its past and future states* (NE, 80)

Changes considered by Leibniz are also dichotomic:

[3] Timelessness of monads and the problem of relation between them and time is analyzed by Mates in (1986).
[4] The Leibnizian idea of individual notion of a substance is considered at first by Russell in (1937, 262). Formally modeled description of this notion was sketched also by Mates in (1968) and this approach was elaborated by Świętorzecka in (2014).

A change is made if … two contradictory propositions are true (AK 6.4.167)

and *immediate* in Aristotelian sense:

> *Change (Mutatio) is an aggregate of two contradictory states. It is necessary, however, that these states be understood as immediate, since between contradictory things a third is not given* (Grua, 323)

To complete the preformal Leibnizian interpretation of LC let us come back to already mentioned feature of time: its relationality. Leibnizian opinion about relational structure of time is explicitly expressed in his many texts and rather often commented in the modern literature (cf. Futch, 2008). His so called reductionism (in contrary to Newtonian substantialism) consists in actually expressed here denying the existence of time (VE VII. 402, D. 268) and just accepting that *time and space are only kinds of order* (VE 115, NE 128). Again it seems that both ideas may be found in Aristotelian lecture (Phys, IV, 11, 219b2-3) which may be considered as a prototype of Leibnizian opposition to concept of absolute time by Newton. The relational nature of time - exhaustively described also by Russell (1937, 126) and Mates (1986, 227) - consists in this that the passage of time may be described only by relations of *being before/after* and *being simultaneous*: events do not occupy some time points but if they are not simultaneous, they are before or after each other. The same relational structure of *space* considered by Leibniz is linked by Russell with relationality of *motion* and this explanation is mentioned to be applied also to change in general:

> *To say that body is at rest* [lasts or is unchanged] *means that its occupancy of certain position in space is simultaneous* [...] *with two events which are not simultaneous with each other. And a body is in motion* [changes] *will mean that its occupancy of one position and its occupancy of another are successive* (Russell, 1937, 128)

Russell claims that relationality of time is not symmetrical to the relationality of space in this sense that:

> *Difference of spatial position at some time shows difference of substance, but difference of temporal position at the same place does not show this. The time-order consists of relations between predicates, the space-order holds between substances"* (Russell, 1937, 128)

However insofar changes are transitions in possessing/non possessing attributes that are expressed just by predicates, the relational character of change also could be "inferred" from the mentioned *time-order*, so actually from the fact that time is reducible to (temporal) notions of simultaneousness and succession. In the presented approach we will link the first of mentioned relation with the specific Leibnizian notion of *compossibility*:

Compossible is what, with another thing, implies no contradiction (Grua, 325)

Compossible things are] those, one of which being given, it does not follow that the other is negated; or those of which one is possible, the other being assumed (AK 6.2.498)

Leibniz explicitly used the notion of compossibility to speak about *many possible Universes* (e.g. G.III. 573)[5] but our intuitions are going now in another direction. Following the quoted conception of Russell we would say now that changeability of any state requires to refer to some other compossible states.

To describe our intuitions in some simpler way let us take an elementary state of monad m_1 named α_1. This can be actual or not and so it is α_1 or $\neg\ \alpha_1$. To speak about a changeability of this state we refer to another one named α_2 – a sentence that is the description of some state of monad m_2 that is compossible with α_1. By this referring we can reconsider the value of α_1 but this is a new stage in which α_1 or $\neg\alpha_1$ is compossible with α_2 or $\neg\alpha_2$. If the value of α_1 is now different from the initial stage we speak about change of α_1. In general if each of the propositional constants $\alpha_1, ..., \alpha_n$ is true or false on stage n, we consider them in relation to α_{n+1} to come to the next stage n+1 on which now each of $\alpha_1, ..., \alpha_{n+1}$ is true or false. This proceeding from one stage to the next one is a necessary condition for so called *C changes* and is described with the help of the introduction of new propositional constants in the object language. In effect we obtain a family of successively growing languages. We define a language of level n as a language built up from the propositional constants $\alpha_1, ..., \alpha_n$ using the usual classical propositional operators together with a one place operator C. Now a formula belonging to a language of level n belongs also to level k>n. We understand by the minimal level of a formula A (lv(A)) the first level of a language to which A belongs. It is the highest index of a propositional constant occurring in A.

Now let us show how the above intuitions may be expressed by LC.

2. Logic LC. Axiomatization and semantics

As we announced we express our logic using the notion of *a language of level n* defined as the set of formulae built up from propositional constants out of the set $\{\alpha_1, ..., \alpha_n\}$ by classical operators $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ and a one place operator C to be read as *it changes, that ...*

LC is characterized by all tautologies of classical propositional logic and all formulae of the following schemata:

(Ax1) $CA \rightarrow C\neg A$

(Ax2) $C(A\wedge B)\rightarrow CA\vee CB$

(Ax3) $\neg A\wedge B\wedge CA\wedge\neg CB\rightarrow C(A\wedge B)$

[5] This context was elaborated especially by Mates and also discussed by Rescher in view of the questionable transitivity of compossibility.

(Ax4) $\neg A\wedge\neg B\wedge CA\wedge CB\rightarrow C(A\rightarrow\neg B)$

and as primitive rules:

(MP) $A, A\rightarrow B \vdash B$; (¬C-rule) $A \vdash \neg CA$; (Rep) $A[B], B\leftrightarrow B' \vdash A[B']$

Now we describe LC semantics.

Let φ be a function from the set of natural numbers to subsets of the set of propositional constants. Let the expression $\varphi\vDash^n A$ be read as *the formula A is true at stage n in some history* φ.

We define the relation $\varphi\vDash^n A$ for $n\leq lv(A)$:

Definition ($\vDash$). For any formula α_k (where 1≤k\≤n):

(i) $\varphi \vDash^n \alpha_k$ iff $\alpha_k \in \varphi(n)$,

Let A, B be formulas of level n, then:

(ii) $\varphi\vDash^n\neg A$ iff $\varphi\nvDash^n A$,

(iii) $\varphi\vDash^n A\wedge B$ iff $\varphi\vDash^n A$ and $\varphi\vDash^n B$,

(iv) – (vi) for $A\vee B$, $A\rightarrow B$, $A\leftrightarrow B$ as usual

(vii) $\varphi\vDash^n CA$ iff ($\varphi\vDash^n A$ and $\varphi\nvDash^{n+1} A$) or ($\varphi\nvDash^n A$ and $\varphi\vDash^{n+1} A$)

If $n < lv(A)$ then $\varphi\vDash^n A$ is not defined.

An example of a history φ is the function $\varphi^*(n)=\{\alpha_n\}$ which codes so called *history of existential transformations* of subsequent individual substances a_1, a_2, a_n,.... Subsequent individual constants α_1, α_2, α_3, ... are situational counterparts of a_1, a_2, a_n, ...

There could be considered histories with different special *rhythms* of changes. Some of them may code Parmenidenian theory of impossibility of change, others e.g. theory of permanent changes of elementary situations. There are no histories which code global changes i.e. there are no histories that on every stage every formula changes its truth value (cf. Świętorzecka 2009).

Our idealization of Leibniz's changes is mentioned to code all possible sequences of this type. Of course all possible histories respect consistency in this sense that every classical propositional tautology is true on every stage of every history.

We continue with the following definitions:

Definition (φ-validity). For any formula A with lv(A) = n: A is φ-valid iff $\varphi \vDash^k A$ for all k≥n.

Definition (validity). For any formula A: A is valid iff A is φ-valid for all functions φ.

The connection of LC and described semantics is described by:

Completeness Theorem. A is valid iff A is derivable in LC

This is shown in (Świętorzecka 2008).

3. Extension of LC by the notion of constancy (□)[6]

To introduce the usual symbol □ for expressing unchangeability we define one place operators $(uC)^n$:

Def. (uC^n). $(uC)^0 A \leftrightarrow A$

$(uC)^{n+1} A \leftrightarrow (uC)^n A \wedge \neg C(uC)^n A$

$(uC)^k$ fulfills the semantic condition:

(viii) $\varphi \vDash^n (uC)^k A$ iff $\forall_i (n \leq i \leq n+k \Rightarrow \varphi \vDash^i A)$

The meaning of the symbol □ is *stronger* than $(uC)^k$:

(ix) $\varphi \vDash^n \Box A$ iff $\forall_{i \geq n}\ \varphi \vDash^i A$

Our definitions allow to notice that:

Fact 1.(a) The following formulae are valid:

(Ax5) $\Box A \rightarrow (uC)^n A$ for all n

(Ax6) $\Box A \rightarrow \neg C \Box A$

(b) The following rule preserves validity of formulas:

(ωr) $B \rightarrow (uC)^n A\ \forall_{n \geq 0} \vdash B \rightarrow \Box A$

(Proofs indirect)

To obtain LC□ we add to LC as axioms (Ax5) and (Ax6) and (ωr).

Fact 2. The following rules do not preserve validity of formulas:

[6] This part of the presented considerations is mentioned to announce a detailed lecture in (Czermak, Świętorzecka, 2013).

(*) $A \to B \vdash CA \to CB$ (**) $A \to B \vdash CB \to CA$

To find *a bridge* between LC□ and some known temporal logics let us notice that:

Fact 3. □ has S4 properties in LC□, e. i. the following fomulas are valid:

(T) $\Box A \to A$

(K) $\Box (A \wedge B) \leftrightarrow \Box A \wedge \Box B$

(4) $\Box A \to \Box\Box A$

(Proofs are inductive with the use of specific Ax5, Ax6 and ωr.)

S4-properties are possessed by □ also in frame of propositional linear temporal logic PTL. Actually it comes out that:

Fact 4. In frame of LC□ extended by a definition of temporal operator N (next) :

(DefN) $NA \leftrightarrow A \leftrightarrow \neg CA$

we obtain all specific axioms of a □○-fragment of linear temporal logic in Goldblat's version (1992) i.e. there are drivable the following formulas:

(*K*) $\Box(A \to B) \to (\Box A \to \Box B)$

(K_N) $N(A \to B) \to (NA \to NB)$

(Fun) $N\neg A \leftrightarrow \neg NA$

(Mix) $\Box A \to A \wedge N\Box A$

(Ind) $\Box(A \to NA) \to (A \to \Box A)$

and also rules:

(Nr) $A \vdash NA$ and (□r) $A \vdash \Box A$

To compare mentioned □○-calculus with LC□ we would have to enrich it by a definition of C:

(DefC) $CA \leftrightarrow (A \leftrightarrow N\neg A)$

A question about this connection we leave here opened.

Thanks to Fact 4 we can say that our philosophically motivated LC□ meets the temporal logic which is also a starting point of many modern idealizations of epistemic issues. However LC□ takes the notion of change as a prior to time.

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